Proofs

Theorem (Binomial Theorem)

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ are the *binomial coefficients*

Lemma

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

Proof:

Proof By Induction

Proof By Combinatorics

Example

Find the first 4 terms of $(2x-3)^6$ in ascending powers of x

Example

In the binomial expansion of

$$(1+kx)^6$$

where k is constant, the coefficient of x^3 is twice as large as the coefficient of x^2. Find the value of k

Example

Calculate the coefficient of x^2 in the expansion of $(2x+1)^3(2-3x)^4$

Example

Find the number of pairs of positive integers x, y which solve the equation:

$$x^3 + 6x^2y + 12xy^2 + 8y^3 = 1,000,000$$